THE KENYA POLYTECHNIC

SURVEYING & MAPPING DEPARTMENT

HIGHER DIPLOMA IN LAND SURVEY

END OF YEAR I EXAMINATIONS

NOVEMBER 2006

MATHEMATICS

3 HOURS

INSTRUCTIONS TO CANDIDATES:

You should have the following for this examination:

Answer booklet

Calculator/Mathematical tables

Answer any FIVE of the following EIGHT questions.

All questions carry equal marks and the maximum marks for each part of a question are as shown.

This paper consists of 3 printed pages.

1. (a) Given the function $f(x,y)=\frac{1}{\sqrt{y}}e^{-\frac{(x-t)^2}{4y}}$ show that $\frac{\partial^2 f}{\partial x^2}=\frac{\partial f}{\partial y}$. (6 marks)

(b) (i) Find the approximate change in volume of a cylinder of radius 3cm and height 12cm when radius increases by 0.4cm and height decreases by 0.25cm. (5 marks)

(ii) Given the function $V=\frac{x^2-y^2}{(x^2+y^2)^2}$, show that: $\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{dy^2}=0.$

(9 marks)

2. (a) If $A=\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}. (3 m arks)

(b) (i) Find the inverse of the following matrix by partitioning:

$$B=\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$ (7 marks)

(ii) Find the eigen values and eigen vectors of the matrix

$$A=\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}.$$ (10 marks)

3. (a) Evaluate:

(i) $\int_1^2 \frac{3x^2+2x+2}{(x+1)(x^2+2)}dx$ (ii) $\int \sin^2 2\theta \cos^3 2\theta d\theta$ (10 marks)

(b) Prove that: $\int_0^1\int_0^1 \frac{x-y}{(x+y)^3}dydx=\frac{1}{2}$. (10 marks)

4. (a) Find the value of the Jacobian $\frac{\partial(u,v)}{\partial(r,\theta)}$ where $u=x^2-y^2$, $v=2xy$ and

$x=r\cos\theta,\ y=r\sin\theta.$ (10 marks)

(b) Given the matrices $A(t)=\begin{bmatrix} t & 1 \\ -1 & t^2 \end{bmatrix}$, $B(t)=\begin{bmatrix} 2 & t^2 \\ t^3 & 1 \end{bmatrix}$ show that

$\frac{dB'}{dt}A' + B'\frac{dA'}{dt} = \frac{d}{dt}(AB)'$ (10 marks)

5. (a) Convert the point (-1, 1, $-\sqrt{2}$) from Cartesian to spherical coordinates.

(8 marks)

(b) Given $y=\tan^{-1}\left[\frac{b}{a}\tan x\right]$, show that $\frac{dy}{dx}=\frac{ab\sec^2 x}{a^2+b^2\tan^2 x}$. (12 marks)

6. (a) Find the points of intersection of the two curves $x^2 = 2y$ and $\frac{y^2}{16} = x$.

Sketch the two curves and calculate the area enclosed by them. (11 marks)

(b) For the parabola $y^2 = 15x$, find the coordinates of the centroid of the area bounded by the curve, x-axis and the ordinate x=5. (5 marks)

(c) Obtain the Maclaurin's series for $\sin^{-1} x$ up to the term in x^5. (4 marks)

7. (a) Show that: $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \tan\left\{\frac{\pi}{4}+\frac{\theta}{2}\right\}$ (6 marks)

(b) Solve the equation: $\cos\theta + \sqrt{3}\sin\theta = 1.24$ in the range $0^0 \le \theta \le 360^0$.

(4 marks)

(c) (i) Use Simpson's rule with five ordinates to evaluate $\int_0^{0.4} x^3 e^{-x^2}\,dx$ correct to six decimal places. (6 marks)

(ii) Find: $\int_0^{\frac{\pi}{6}} \theta\cos 5\theta\, d\theta$ (4 marks)

8. Differentiate the following with respect to x:

(i) $y=\frac{\sec x+\tan x}{\sec x-\tan x}$ (ii) $y = x^2\sin\frac{1}{x}$

(iii) $y=\tan^{-1}[\sec x+\tan x]$ (iv) $y=e^{3x}x^4\sin\frac{1}{3}x$

(v) $x^3+y^3+3xy^2=8$ (20 marks)