THE KENYA POLYTECHNIC UNIVERSITY COLLEGE

DEPARTMENT OF SURVEYING & MAPPING

DIPLOMA IN LAND SURVEY

END OF YEAR I EXAMINATIONS

NOVEMBER 2007

MATHEMATICS

3 HOURS

INSTRUCTIONS TO CANDIDATES:

You should have the following for this examination:

Answer booklet

Scientific calculator

Answer any FIVE of the following EIGHT questions.

All questions carry equal marks and the maximum marks for each part of a question are as shown.

This paper consists of **4** printed pages.

1. (a) (i) Make 'W' the subject of the formula if

$$Z = \sqrt{R^2 + \left[WL - \frac{1}{LQ}\right]^2}$$

(ii) Determine the value of W when $Z = \frac{1}{7}$, $R = \frac{1}{3}$, $Q = 0.01$ and

$L^2 = \frac{1}{5}$. (9 marks)

(b) Solve for x in the quadratic equation $4x^2 + 8x - 20 = 0$. (4 marks)

(c) Solve for X, Y and Z in the equations:

$$X + Y + Z = 0$$
$$4^{3Z-2} + 2^Y + 16^{X+2} = 32$$
$$3^Z + 3^{4X+1} + 243^Y = 27$$

(7 marks)

2. (a) A solid pyramid on a square base of side 4cm has the slant side of length 12cm. Find, correct to 2d.p., the:

(i) Vertical height of the pyramid

(ii) Volume of the pyramid (6 marks)

(b) Figure 1 is an annulus frustum having measurements as shown. Use the measurements to calculate the volume of the solid in cm³.

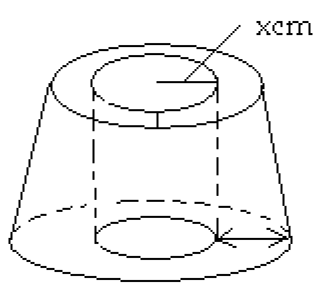

Figure 1

Assume the height of the big pyramid = 4xcm.

(c) Solve for x in the given equation: $4\left(\frac{x}{3} + \frac{1}{4}\right) - 5\left(\frac{x}{3} - \frac{1}{8}\right) = \frac{1}{7}(2x - 16)$

(4 marks)

3. (a) Find the equation of the circle whose diameter has the end-points (4, 3) and (6, 1). (5 marks)

   (b) Determine the equation of a circle passing through the three points 5,3), (6,2) and 3,-1). Give the center and the radius. (10 marks)

   (c) Given the equation $x^2 + y^2 + 6x + 10y - 2 = 0$, find the radius and the center of the circle. (5 marks)

4. (a) Obtain the first four terms of the expansion $\left(1 + \frac{1}{12}x\right)^{10}$ in ascending powers of x; Hence find the value of $(1.005)^{10}$, correct to 4 decimal places. (6 marks)

   (b) Simplify, leaving your answer in surd form: $\left(1+\sqrt{2}\right)^4 - \left(1-\sqrt{2}\right)^4$ (6 marks)

   (c) How many permutations are there of r objects chosen from n unlike objects? (4 marks)

   (d) A mixed hockey team containing 5 men and 6 women is to be chosen from 7 men and 9 women. In how many ways can this done? (4 marks)

5. (a) What is a parabola? (2 marks)

   (b) Solve for the directric line and the focus of a parabola whose equation is given as $2x^2 + 5y - 3x + 4 = 0$. Sketch the parabola on a Cartesian plane. (8 marks)

   (c) Given an ellipse equation $8x^2 + 6y^2 + 64x + 72y + 108 = 0$. Analyze the ellipse and find the major axis. (6 marks)

   (d) Prove if the following equations are orthogonal:

   (i) $x^2 + y^2 - 4x + 2 = 0$ (ii) $x^2 + y^2 + 6y - 2 = 0$

6. (a) Solve the equation: $2.9\cos^2\alpha - 7\sin\alpha + 1 = 0$. (5 marks)

   (b) From the tip of a vertical cliff 80.00cm high the angles of depression of two buoys lying due east of the cliff are 23^0 and 15^0 respectively. How far apart are the buoys? (8 marks)

   (c) Prove that $\sin 3A = 3\sin A - 4\sin^3 A$ (4 marks)

(d) If $SinA = \frac{3}{5}$ and $CosB = \frac{15}{17}$ where A is obtuse and B is acute, find the exact value of $|\ Sin(A+B)\ |$? (3 marks)

7. (a) Find the area of a triangle whose sides are 11.3cm, 9.1cm and 7.8.

(4 marks)

(b) Find the maximum and minimum values of $2\sin\theta - 5\cos\theta$, and the corresponding values of θ between 0^0 and 360^0. (10 marks)

(c) A swimming pool is 55.0m long and 10.0m wide. The perpendicular depth at the deep end is 4.2m and at the shallow end is 140cm, the slope being uniform. The pool needs two coats of a protective paint inside. Find how many litres of paint will be required if a litre covers $12m^2$.

8. (a) Find the approximate value of $\frac{1-\cos 2\theta}{\theta \tan\theta}$ when θ is small. (4 marks)

(b) Differentiate:

(i) $\sin(2x+3)$

(ii) $\cos^2 x$ (6 marks)

(c) Find the equation of the locus of a point P which moves so that it is equidistant from two fixed points A and B whose coordinates are 3,2) and (5,-1) respectively. (5 marks)

(d) Find the equation of the tangent and normal to the curve $y = 3x^2 - 8x + 5$ at a point where $x = 2$. (5 marks)